1.5 Composition of Functions

Sum: $(f+g)(x) = f(x) + g(x)$

difference: $(f-g)(x) = f(x) - g(x)$

product: $(f \cdot g)(x) = f(x) \cdot g(x)$

quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Ex 1 Given $f(x) = x^2 + 2$ and $g(x) = -4x + 7$

① $(f+g)(x) =$

$(x^2+2) + (-4x+7)$

$x^2 + 2 - 4x + 7$

$\boxed{x^2 - 4x + 9}$

② $(f-g)(x) =$

$(x^2+2) - (-4x+7)$

$x^2 + 2 + 4x - 7$

$\boxed{x^2 + 4x - 5}$

③ $(f \cdot g)(x) =$

$(x^2+2)(-4x+7) =$

$-4x^3 + 7x^2 - 8x + 14$

$\boxed{-4x^3 + 7x^2 - 8x + 14}$

④ $\left(\frac{f}{g}\right)(x) =$

$\frac{f(x)}{g(x)} = \frac{x^2+2}{-4x+7}$

Domain: $x \neq \frac{7}{4}$

$-4x + 7 = 0$

$-7 \quad -7$

$\frac{-4x}{-4} = \frac{-7}{-4}$

$x = 7$

Composition of functions — A way of combining functions in which the output of one function is used as the input of another.

$[f \circ g] = f(g(x))$ or $[g \circ f] = g(f(x))$

1st: outside function (f) 2nd: what you plug in (g), so order matters.

Ex. 2. Given $f(x) = x^2 - 1$ and $g(x) = \frac{1}{2x+1}$

Ⓐ Find $(f \circ g)(x)$

outside (f) inside (g)

$\left(\frac{1}{2x-1}\right)^2 + 1$ ← Simplify if possible

Domain $\sqrt{(2x+1)^2} = \sqrt{0}$

$2x - 1 = 0$

$+1 \quad +1$

$\frac{2x}{2} = \frac{1}{2}$

$x \neq -\frac{1}{2}$, $\left(\frac{1}{2x+1}\right)^2 + 1 = (f \circ g)(x)$

Ⓑ Find $(g \circ f)(x)$

$\frac{1}{2(x^2-1)+1} = \frac{1}{2x^2-2+1} = \frac{1}{2x^2-1}$

Domain = $2x^2 - 1 = 0$

$+1 \quad +1$

$\frac{2x^2}{2} = \frac{+1}{2}$

$x^2 = -\frac{1}{2} \Rightarrow x \neq \frac{\sqrt{1}}{\pm\sqrt{2}}$

Ex. 3 Given $f(x) = x^2 + 2$ and $g(x) = -4x + 7$

Find $(f+g)(-3)$

$x^2 + 2 + (-4x + 7)$

$x^2 - 4x - 9$ ① Simplify

$(-3)^2 - 4(-3) - 9$ ② Evaluate at -3 (plug in -3)

$9 + 12 - 9$

$\boxed{12}$