1.6 Inverse Functions 10/2-5/15

Inverse Function – the result of exchanging the input and output values

$$f(x) \Rightarrow f^{-1}(x)$$

One-to-one – a function is one-to-one if the function's inverse is also one-to-one

Graphing an inverse – to graph an inverse flip (x, y) to (y, x).

*A Graph and its inverse reflects across the line $y = x$.

Ex (1) – Graph $f(x) = x^2$ and find $f^{-1}(x)$ (Check table on calculator)

TABLE (Numerically)

$f(x)$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$\Leftrightarrow$ Inverse of each other

$f^{-1}(x)$

x	y
9	-3
4	2
1	-1
0	0
1	1
4	2
9	3

Ex. 2. Find the inverse of $f(x) = 3x - 2$.

Algebraically

$f(x) = 3x - 2$

$y = 3x - 2$

$x = 3y - 2$

$+2 \qquad +2$

$\frac{x+2}{3} = \frac{3y}{3}$

$\frac{x+2}{3} = f^{-1}(x)$

Steps:
1. Change $f(x)$ to y.
2. Switch x & y.
3. Solve for y.
4. Change y to $f^{-1}(x)$

* To check: $f(f^{-1}(x)) = x$, must be true

$f(x) = 3x - 2$ $\qquad f^{-1}(x) = \frac{x+2}{3}$

$f(f^{-1}(x)) = x$ $\qquad f^{-1}(f(x)) = x$

$3\left(\frac{x+2}{3}\right) - 2 = x$

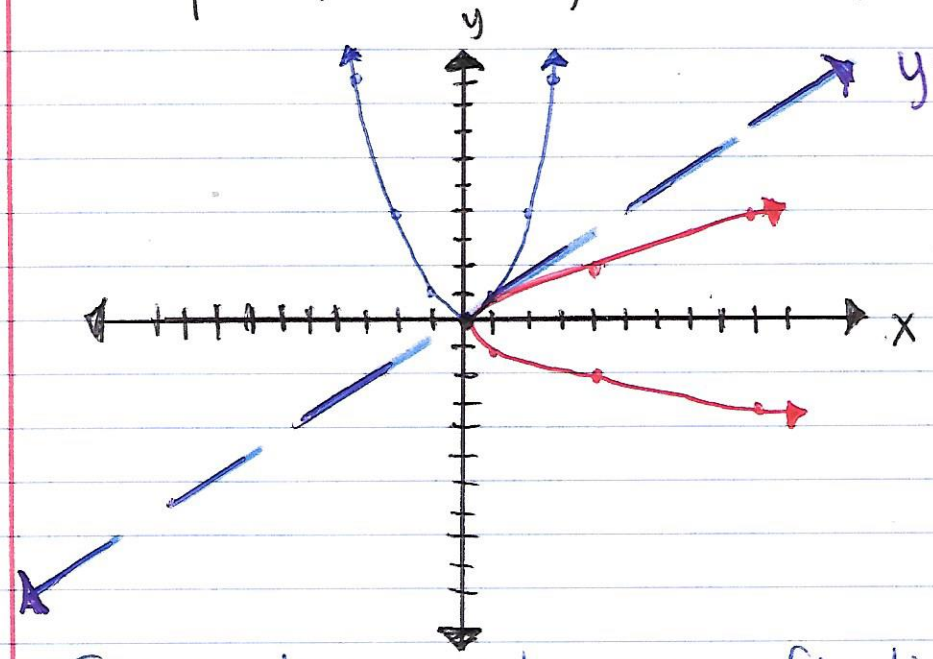
$x + 2 - 2 = x$

$x = x$ ✓ checks out

Therefore, $f(x) = 3x - 2$ & $f^{-1}(x) = \frac{x+2}{3}$ are inverses of each other.

Ex ③ Graph $f(x) = x^2$, and its inverse.

Graphing



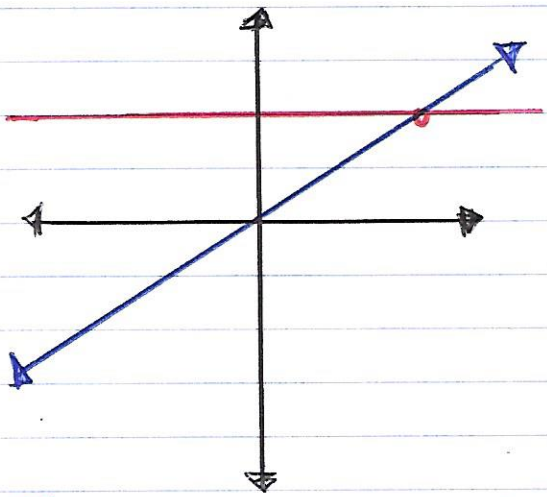
In order for these graphs to be inverses of each other, they must reflect across the line $y=x$

Since the graphs are reflecting across $y = x$, yes the are inverses of each other.

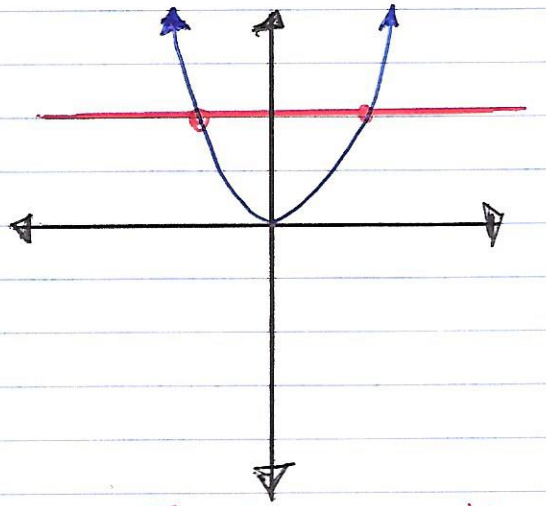
Ex ④ - Determine whether the function is one-to-one.

One-to-one
x's and y's don't repeat

* Use Horizontal Line Test : HTL



Touches graph onces, yes it's one-to-one x's & ys do not repeat



Touches more than once, Not one-to