3.3 Properties of Logs

If using old calculators (Not N-spire):

Change-of-Base Formula

Let a & $b \in \mathbb{R}$ s.t $a \neq 1$ & $b \neq 1$

Base b	Base 10	Base e
$\log_a x = \frac{\log_b x}{\log_b a}$	$\log_a X = \frac{\log_{10} X}{\log_{10} a}$	$\log_a x = \frac{\ln x}{\ln a}$

Ex. 1

a) $\log_4 25 = \frac{\log_{10} 25}{\log_{10} 4} \approx \frac{1.39794}{0.60206} \approx 2.32$

Natural log

b) $\log_4 25 = \frac{\ln 25}{\ln 4} \approx \frac{3.21888}{1.38629} \approx 2.32$

Properties of logs

1. Product Property: $\log_a(u \cdot v) = \log_a u + \log_a v$
2. Quotient Property: $\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$
3. Power Property: $\log_a u^n = n \cdot \log_a u$

Ex 2. Write each log in terms of $\ln 2$ & $\ln 3$

a) $\ln 6$

$\ln 6 = \ln(2 \cdot 3)$
$= \ln 2 + \ln 3$

b) $\ln \frac{2}{27}$

$\ln 2 - \ln 27$
$\ln 2 - \ln 3^3$
$\ln 2 - 3 \ln 3$

Ex. 3. Use properties of logs to verify that $-\log_{10}\frac{1}{100} = \log_{10} 100$

$$-\log_{10}\frac{1}{100} = -\log_{10}\frac{1}{100^1}$$

$$= -\log_{10} 100^{-1}$$

$$= (-1)(-\log_{10} 100)$$

$$= \log_{10} 100$$

Ex. 4. Use properties of log to expand each expression.

a) $\log_4 5x^3y = \log_4 5 + \log_4 x^3 + \log_4 y$

$$= \log_4 5 + 3\log_4 x + \log_4 y$$

b) $\ln \frac{\sqrt{3x-5}}{7} = \ln \frac{(3x-5)^{1/2}}{7}$

$$= \ln (3x-5)^{1/2} - \ln 7$$

$$= \frac{1}{2}\ln(3x-5) - \ln 7$$

Ex. 6 Use properties to condense each expression

a) $\frac{1}{2}\log_{10} x + 3\log_{10}(x+1)$

$\log_{10} x^{1/2} + \log_{10}(x+1)^3$

$\log_{10}\sqrt[2]{x^1} + \log(x+1)^3$

$\log_{10}[\sqrt{x}\cdot(x+1)^3]$

b) $\frac{1}{3}[\log_2 x + \log_2(x-4)]$

$\frac{1}{3}[\log_2(x\cdot(x-4))]$

$\log_2[x(x-4)]^{1/3}$

$\log_2\sqrt[3]{x(x-4)}$

p. 206 #7-23 odd 37-43 odd, #47-63 odd 69-83 odd

#93-105

19 problems